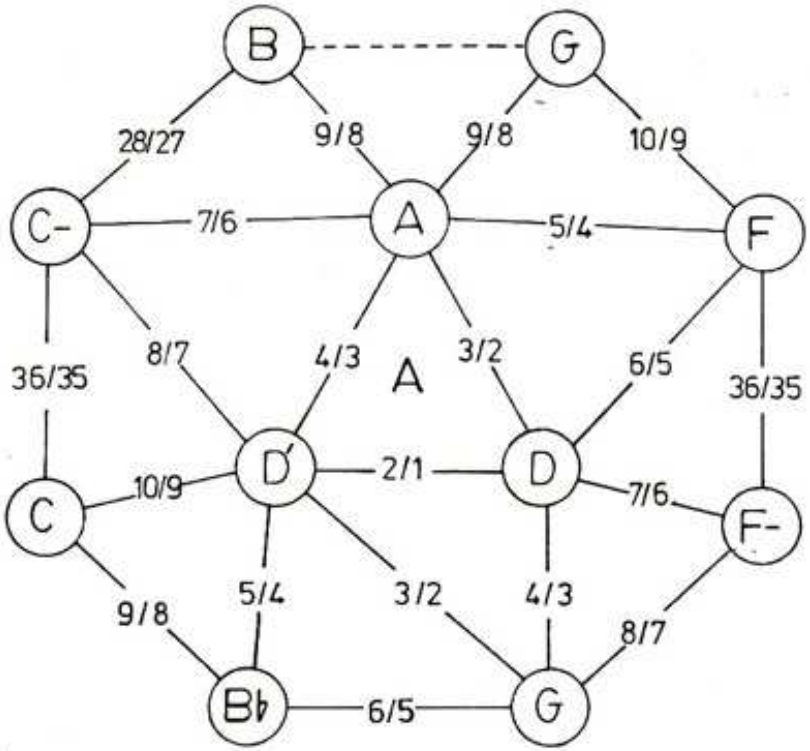
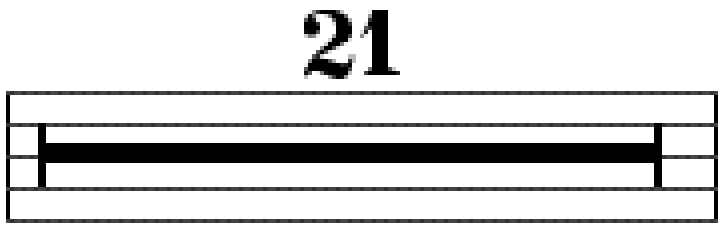
Numbers & Music

What Is The Connection?



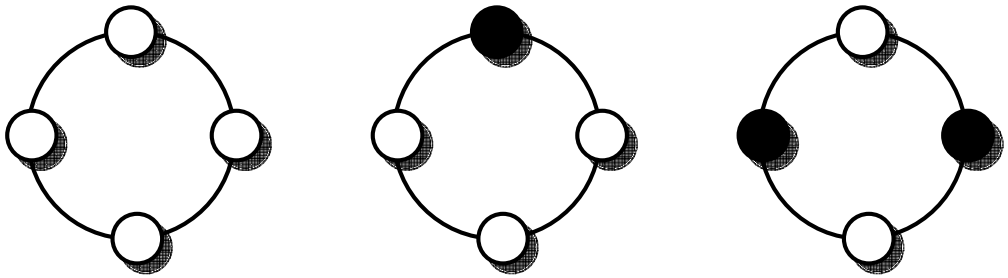
Counting

- "Nothing can be farther from the working musician's mind than counting, nothing farther from the working mathematician's mind than singing, and yet there is something common to both." — Viktor Zuckerkandl, *Man the Musician*, 1973



Counting Necklaces

- How many 4-bead necklaces?
- Two possible colors for each bead



First Answer

- $2^4 = 16$ variations

- But some of these are the same!

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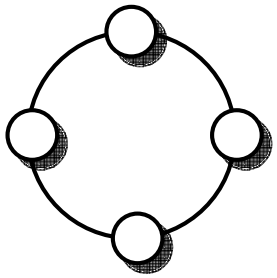

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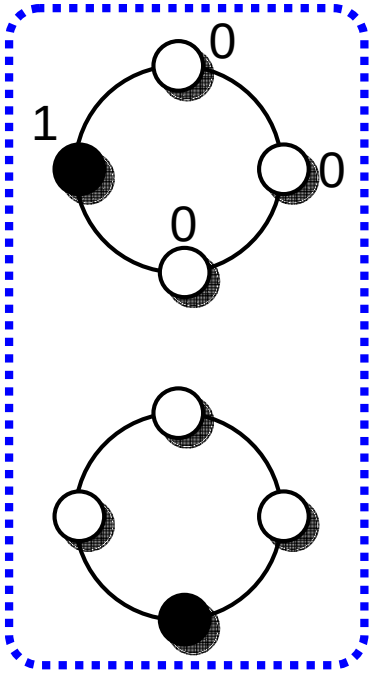

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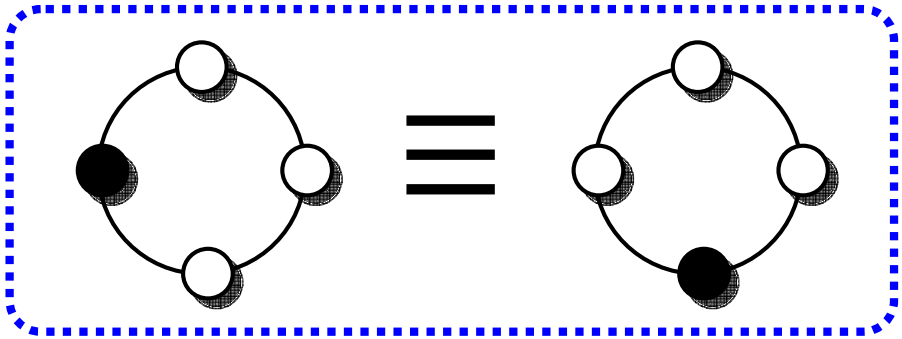
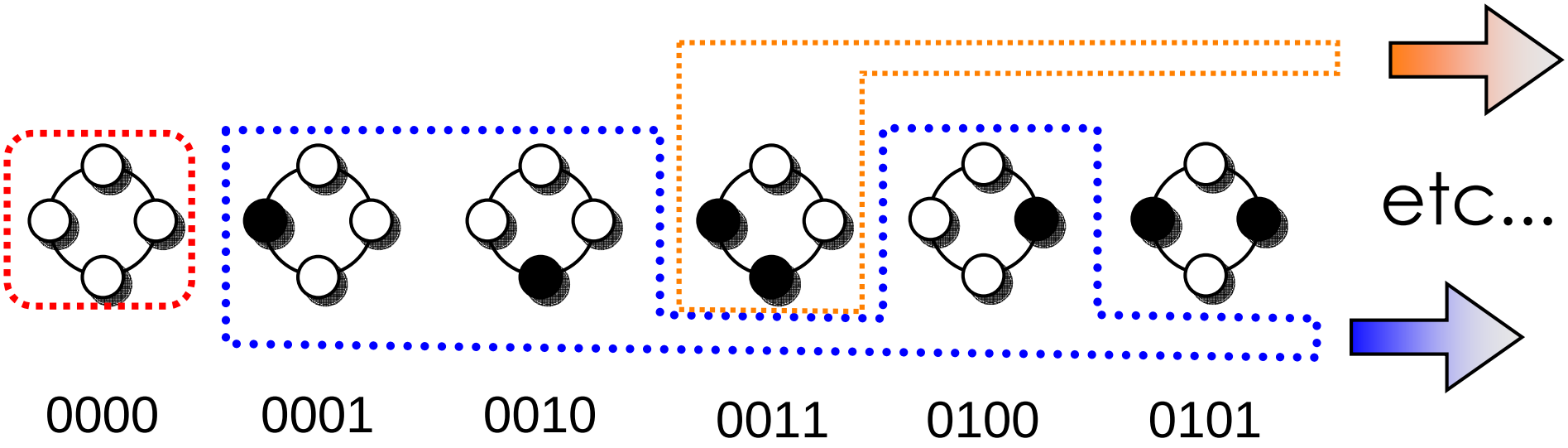
Equivalence

- Necklaces are equivalent by rotation
- Equivalence relation ⇒ a partition into classes



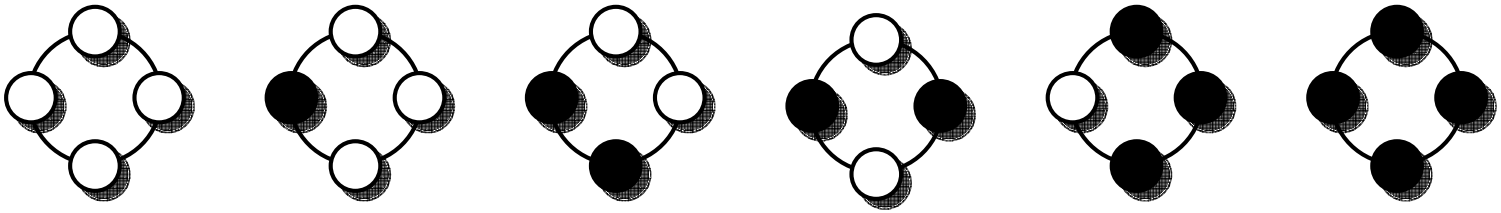
Classes

- Start by listing them all
- Circle the equivalent graphs



The Answer

- There are 6 different necklaces



Patterns

- Patterns are easier to see with more dimensions
- Example: roots of polynomials in the complex plane

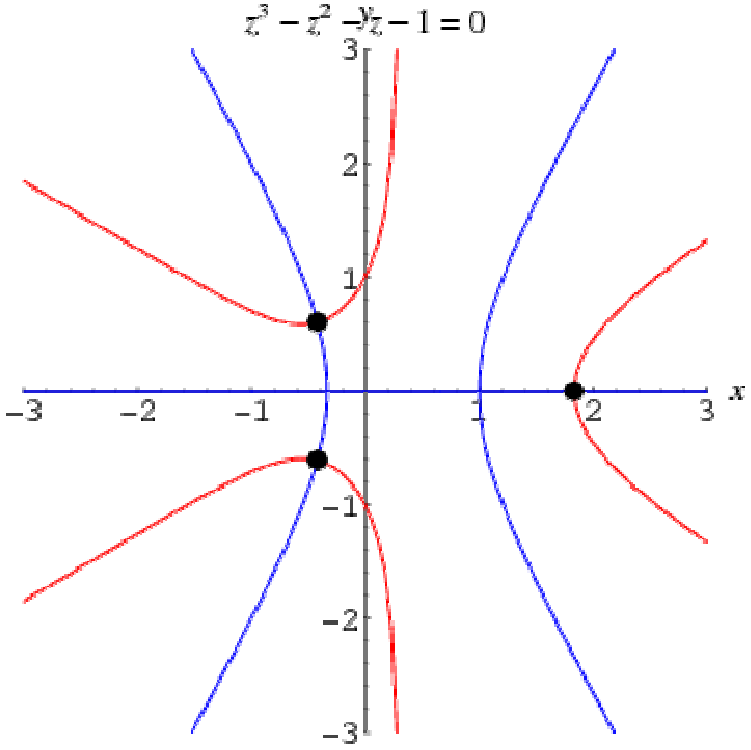
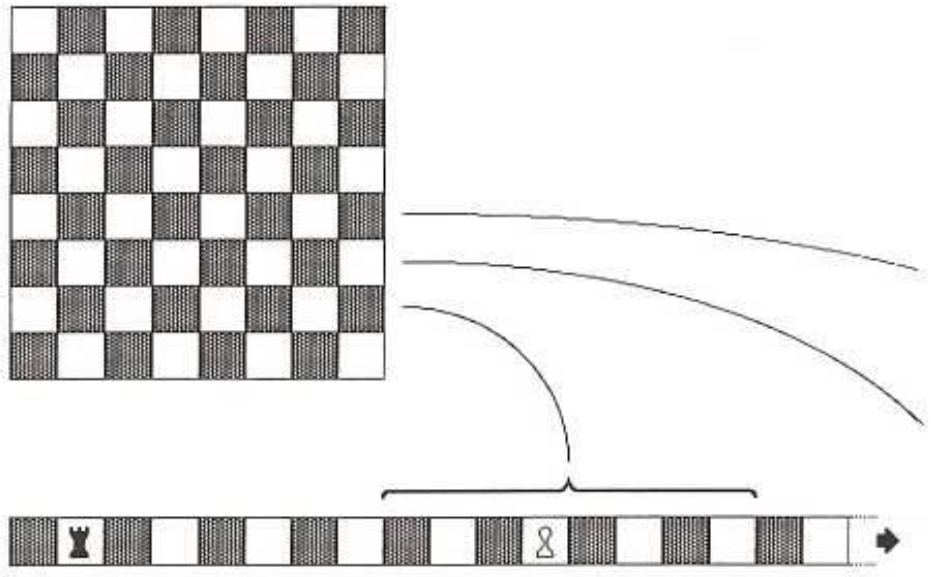


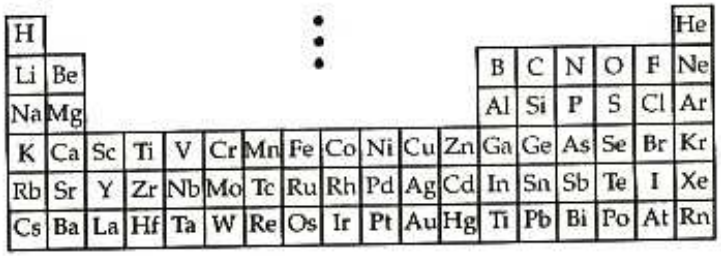
Table *vs.* List

- Is the pawn in danger?
- How would a rook move?
- How would a knight move?



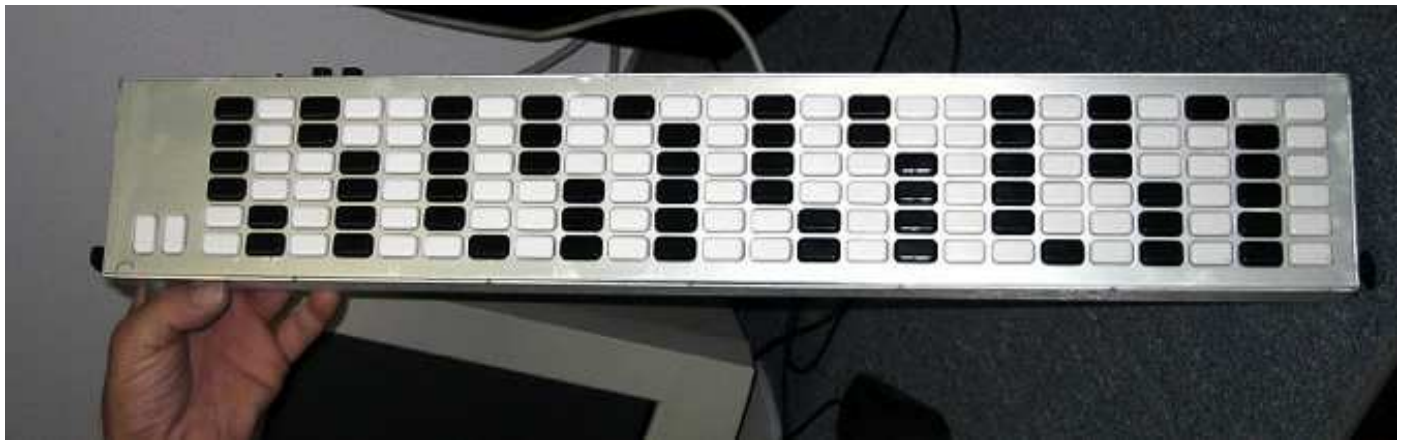
Менде
ле́ев

1 H Hydrogen
2 He Helium
3 Li Lithium
4 Be Beryllium
5 B Boron
6 C Carbon
7 N Nitrogen
8 O Oxygen
9 F Fluorine
10 Ne Neon



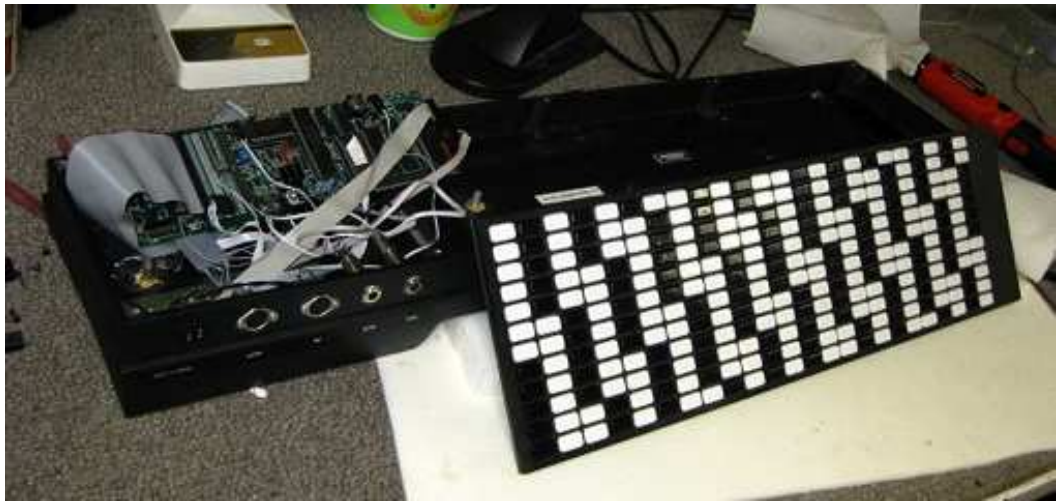
31 Ga Gallium
32 Ge Germanium
33 As Arsenic
34 Se Selenium
35 Br Bromine
36 Kr Krypton

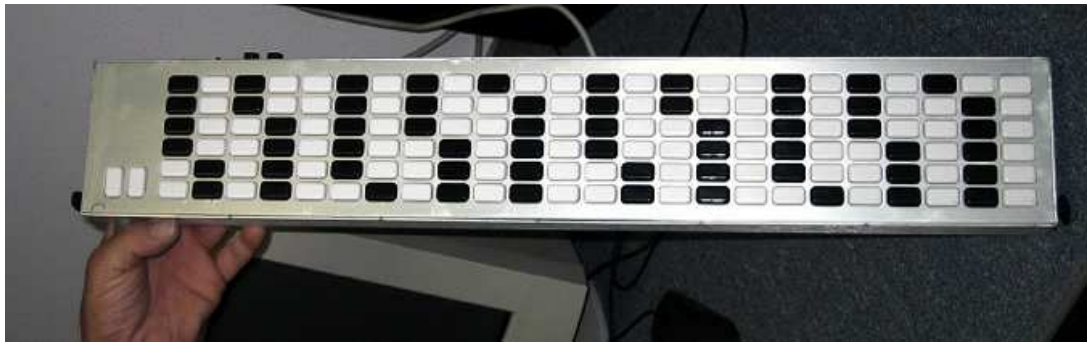
Piano, Guitar, ZBoard



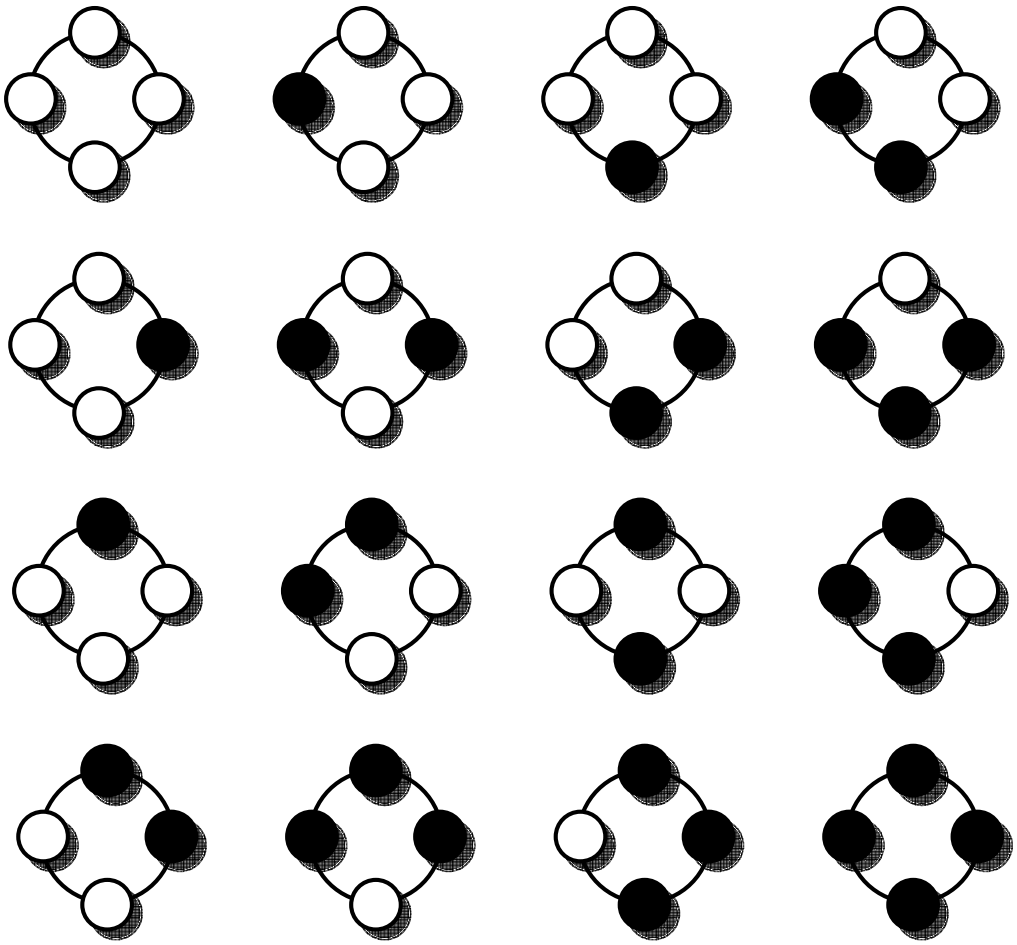
ZBoards & ZTars

- Built by Harvey Starr in San Diego

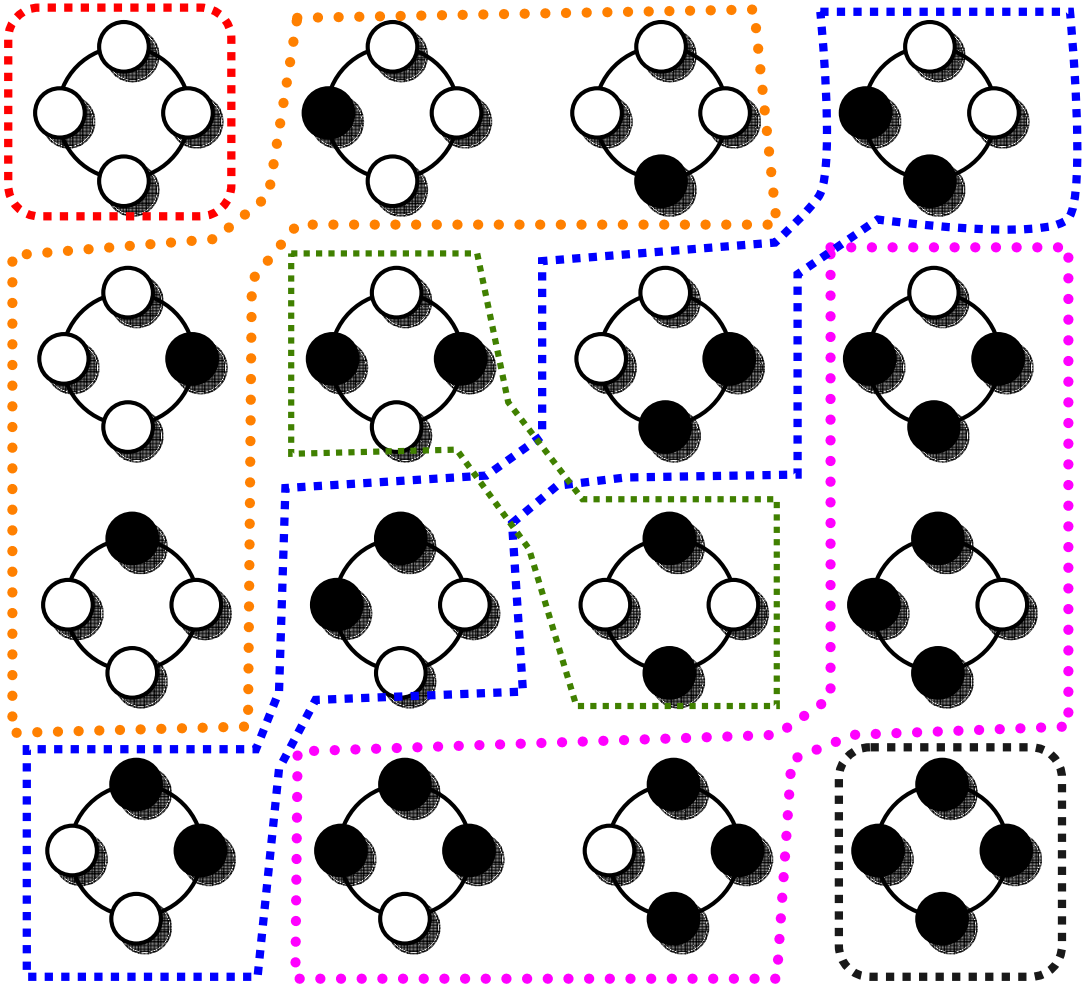




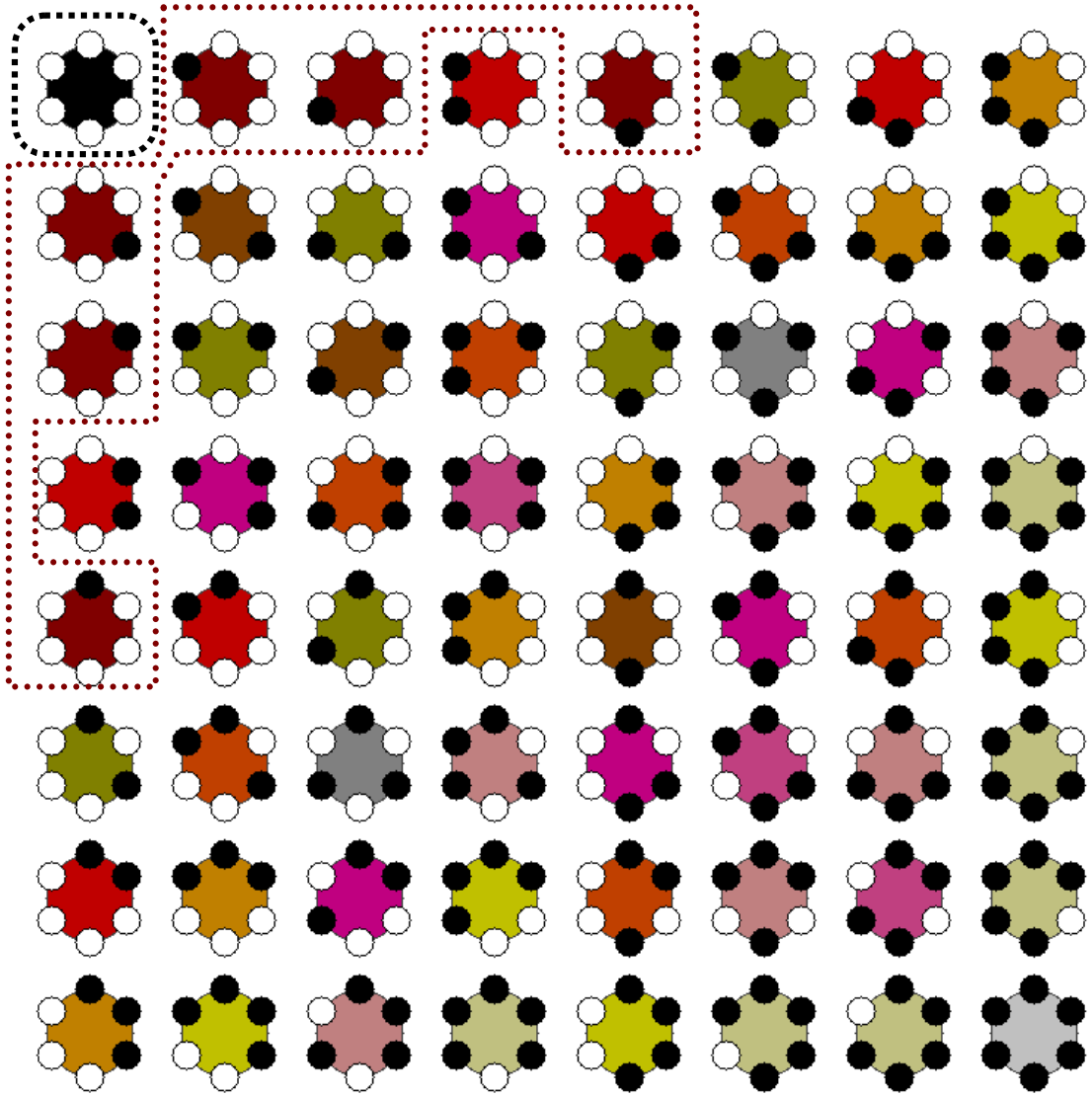
All 16 Species



All 16 Species



N = 6

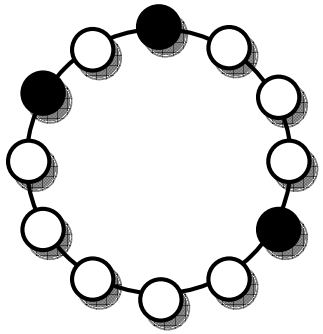


10 01 11

R G B

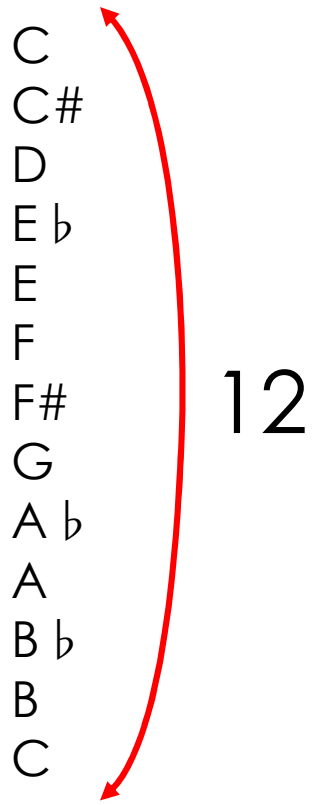
More Complicated

- Now we look at "necklaces" with 12 beads
- Many more cases!
- Why 12?



Well-Tempered

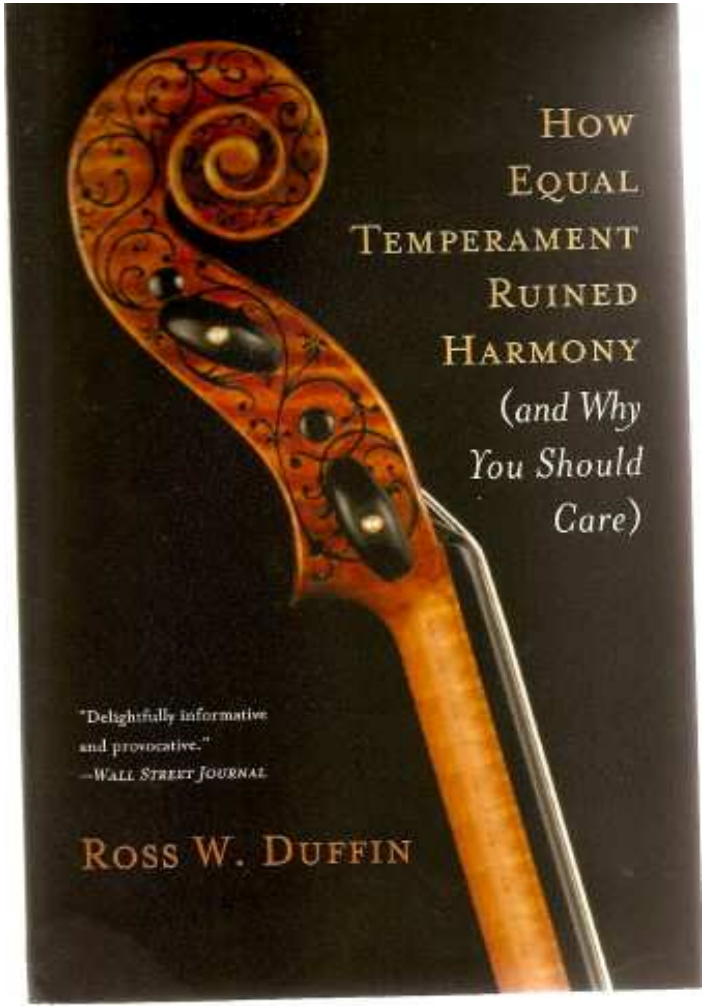
- Western music divides octave into twelve equal step
- Is it just luck that this works so well?
- $(3/2)^{12} = 129.7... \approx 128$
- Twelve perfect fifths takes you (almost) through seven octaves



Is This Good?

- The perfect fifth is too flat
  - $2^{7/12} = 1.498... \approx 3/2$
- The major third is too sharp
  - $2^{4/12} = 1.26... \approx 5/4$

Maybe Not


How Equal Temperament Ruined Harmony (and Why You Should Care)
"Delightfully informative and provocative."
—Wall Street Journal
Ross W. Duffin

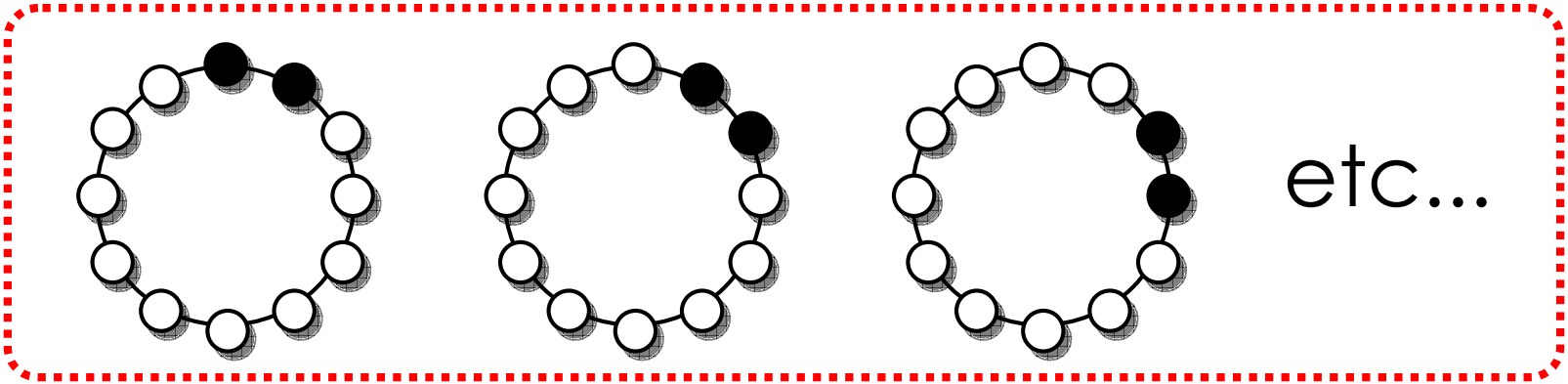
Conductor's Complaint

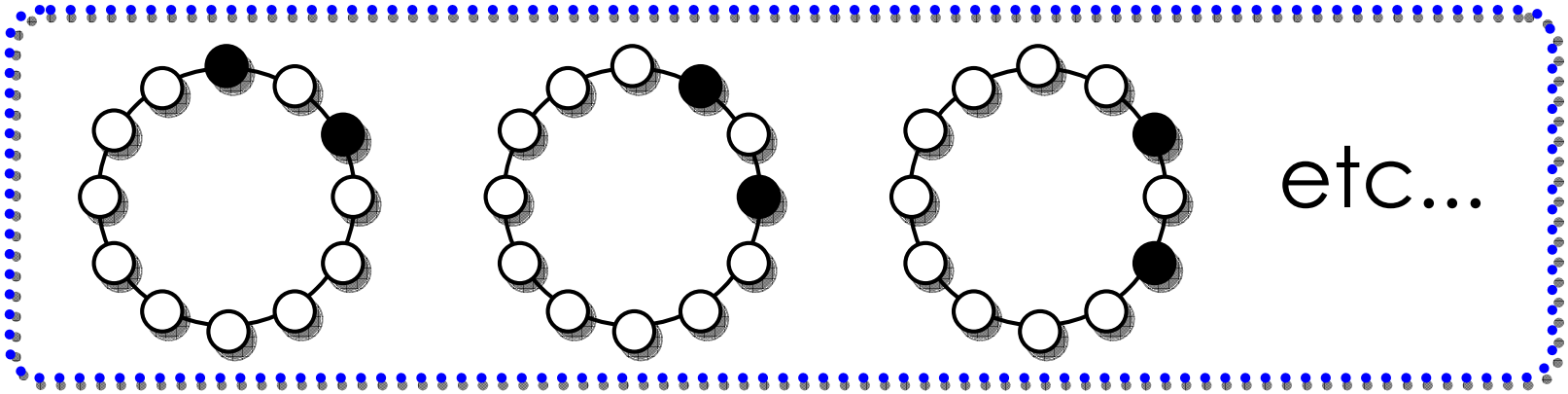
- Christoph von Dohányi talks about Beethoven's Ninth Symphony

> The symphony begins with about two minutes of a D-minor chord. But after that D minor comes a striking shift to B-flat major. In rehearsal, I just couldn't get that B-flat chord to sound right. I mean, I know what a major third is, and all of the players are consummate professionals, but we tried it over and over and I was never satisfied.

- A B-flat major chord needs a slightly flat D to sound sweet

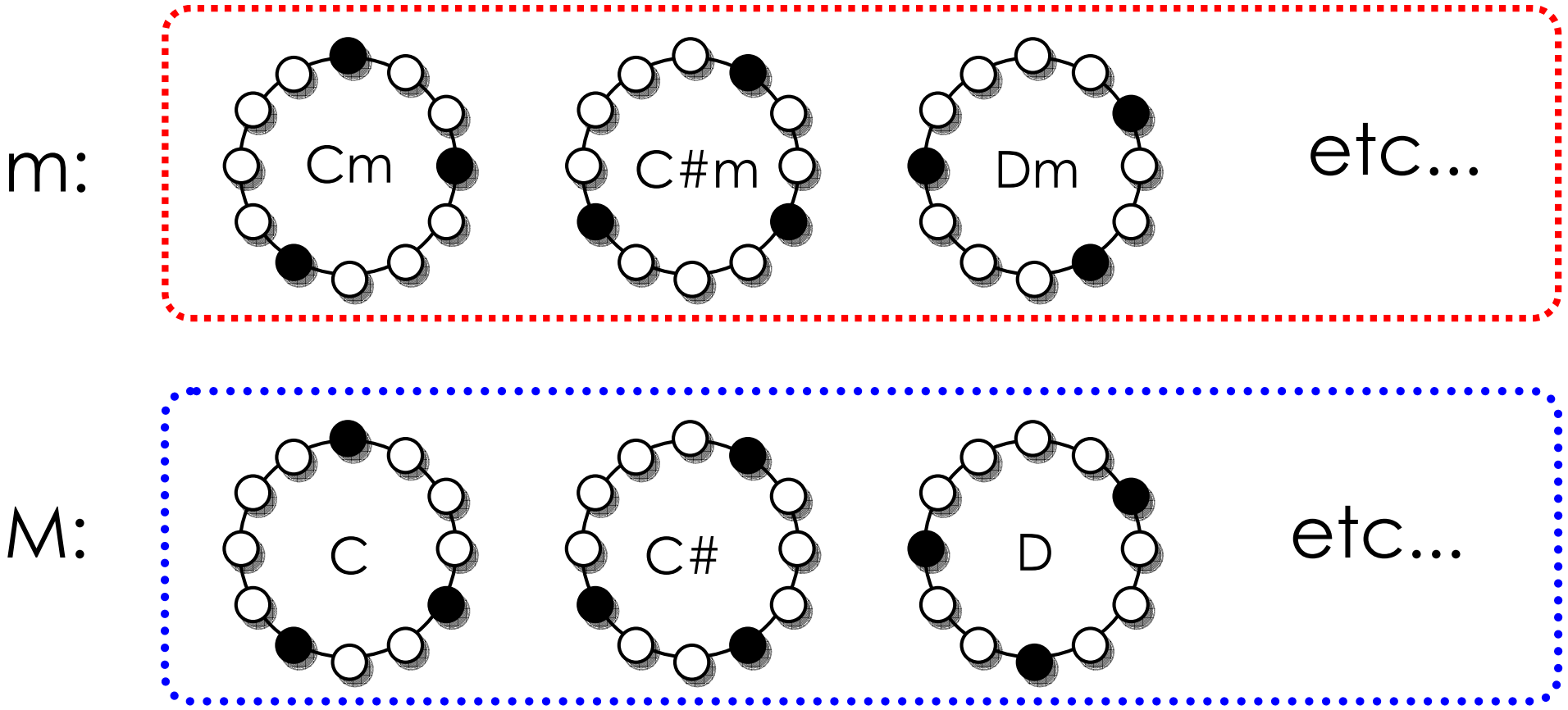
Intervals

m2/M7:  etc...

M2/m7:  etc...

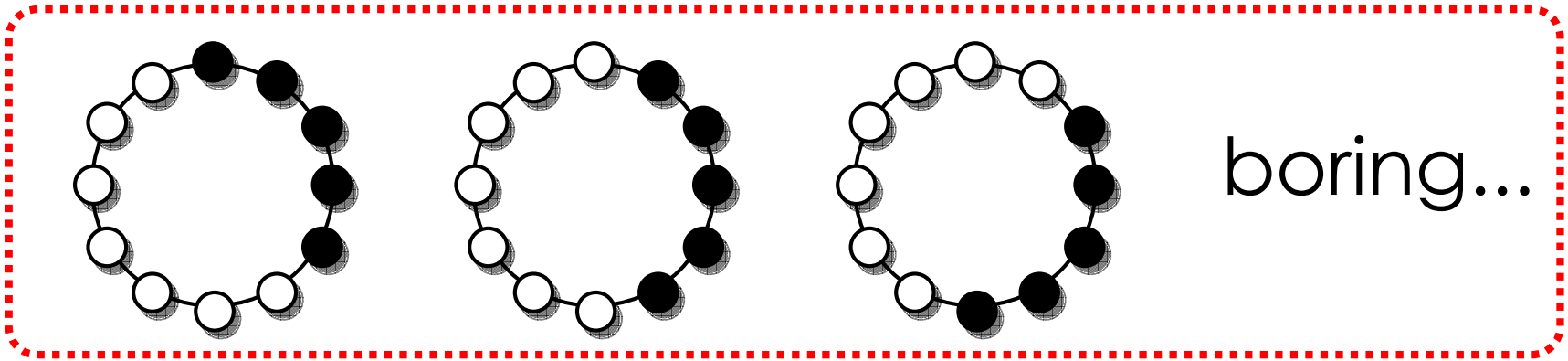
There are 6 different intervals

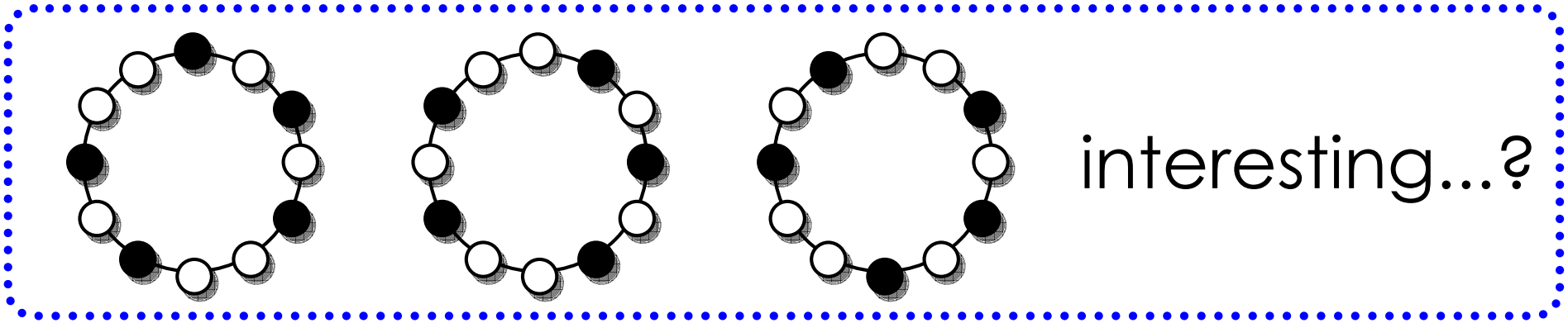
Triads (Chords)



There are 19 of these classes

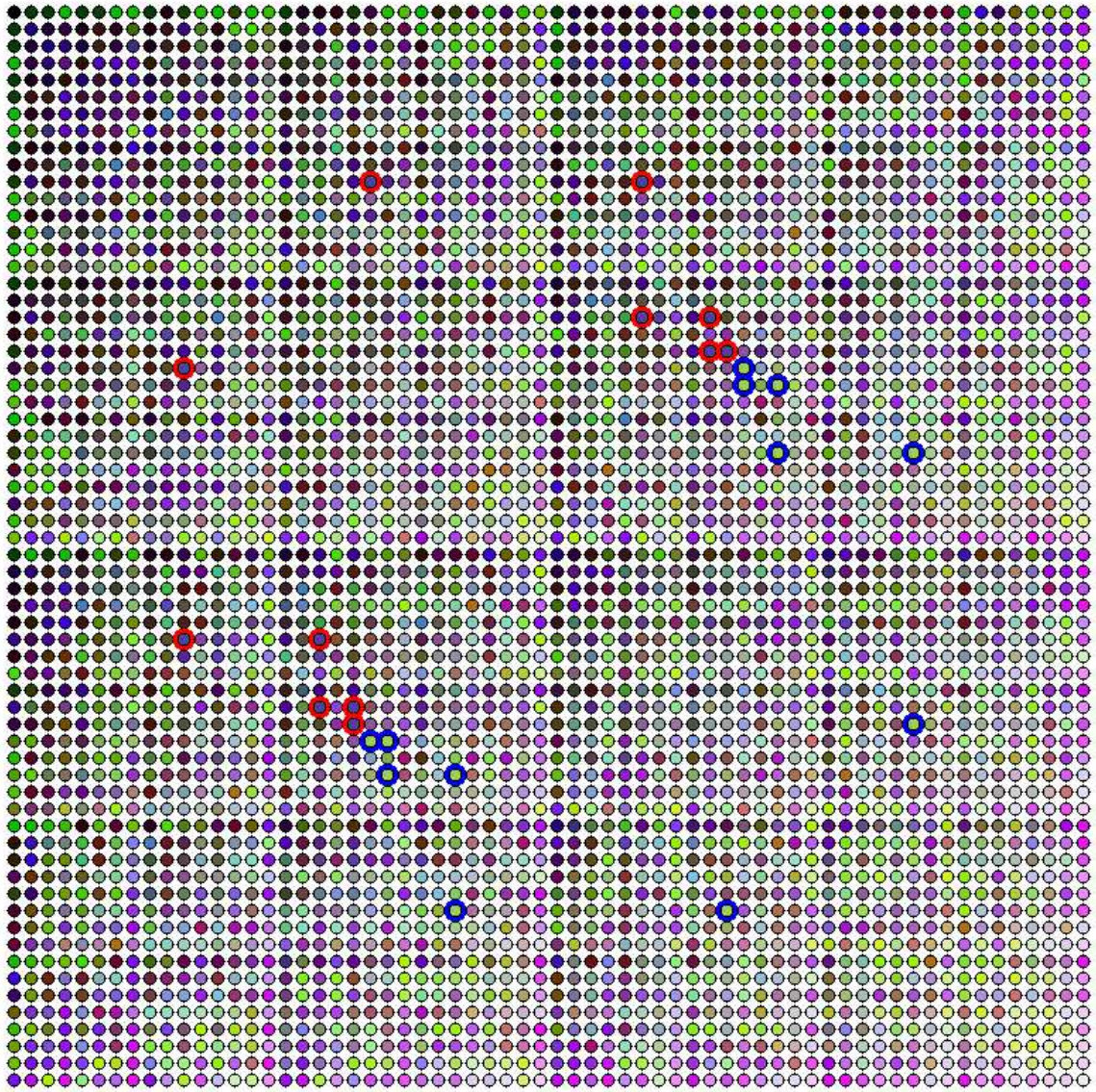
Scales





There are 66 different 5-note scales

All 4096 Patterns



pentatonic

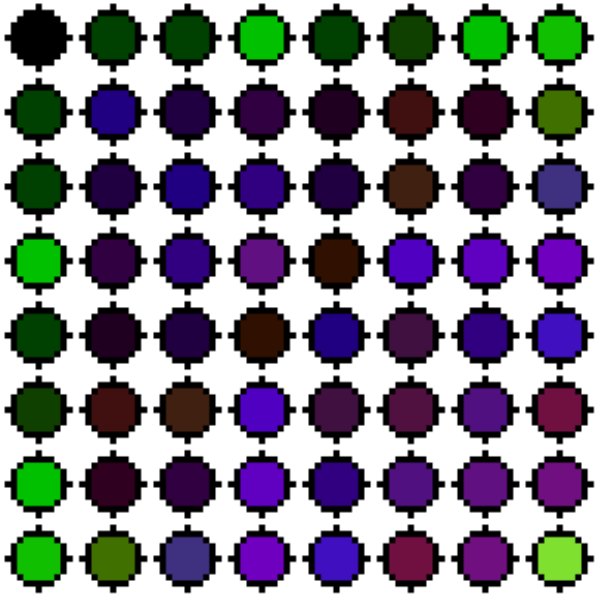
diatonic

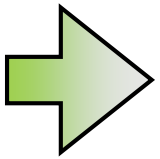
1010 1101 0101

R G B

352 distinct classes/colors

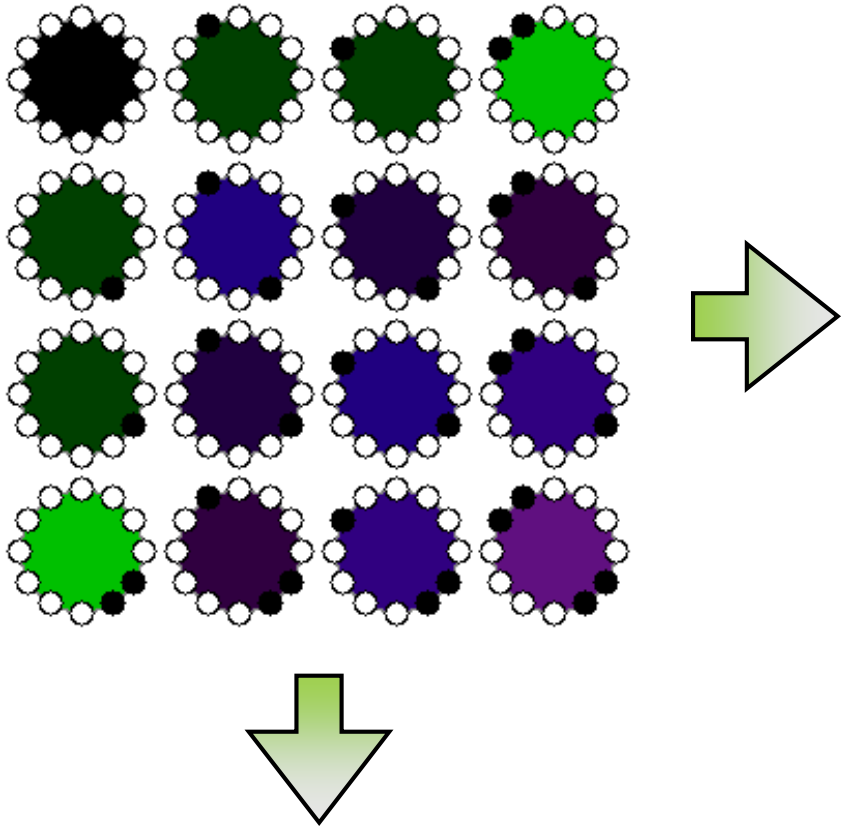
A Closer Look





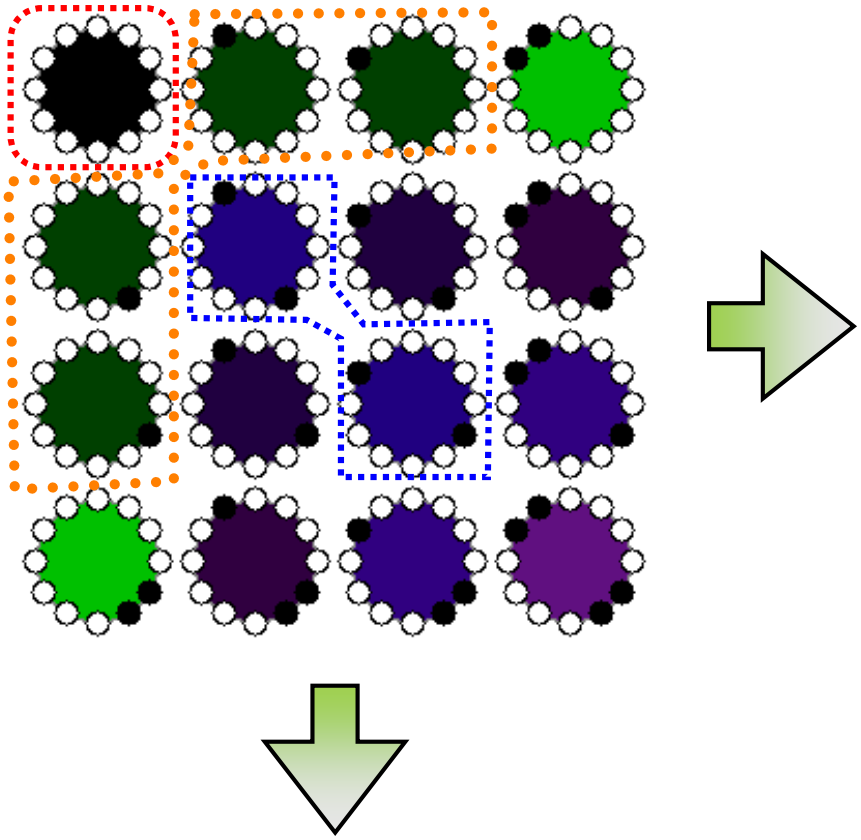
- Full image at http://www.andrewduncan.ws/cmt/graphs_small.png

Even Closer



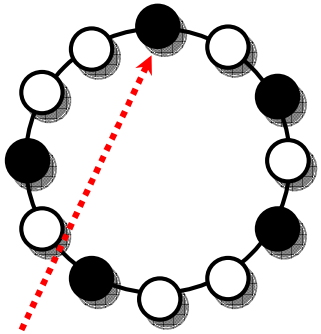
- http://www.andrewduncan.ws/cmt/graphs_big.png

Similar Patterns



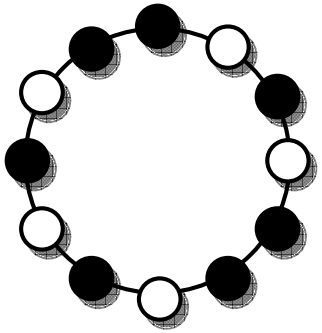
- But on a larger scale!

Favorites



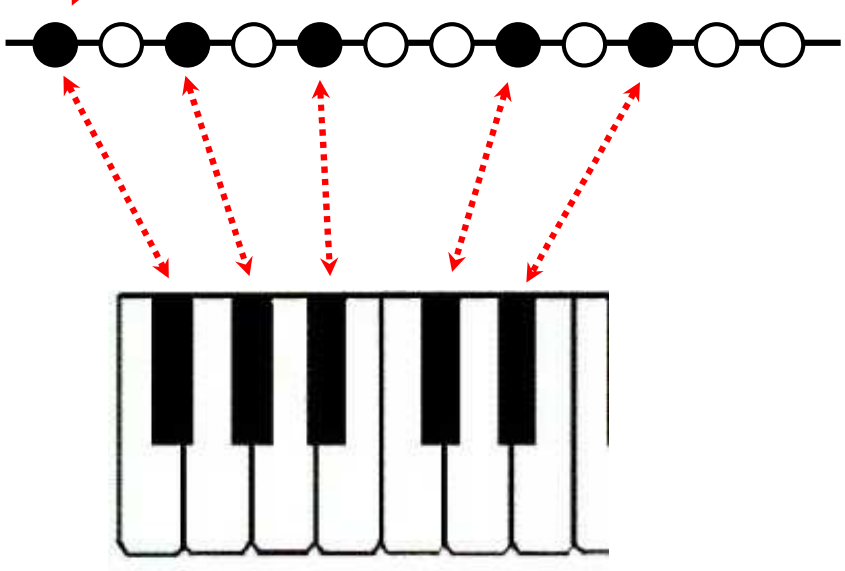
complement!

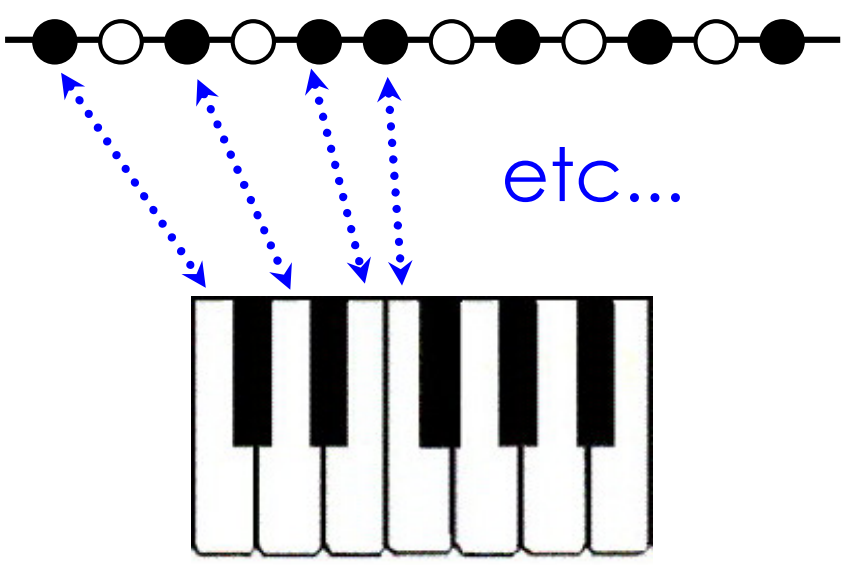
obvious?



pentatonic

diatonic





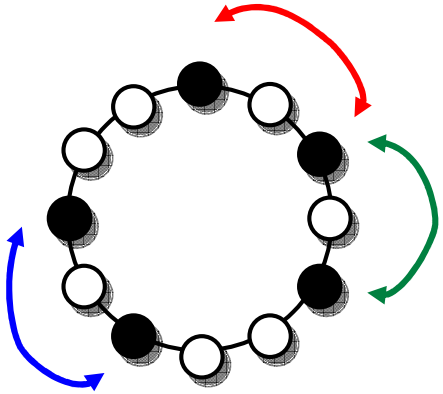
etc...

Why?

- These scales seem to contain many patterns
- How to describe this?
- We look at interval content

Interval Content

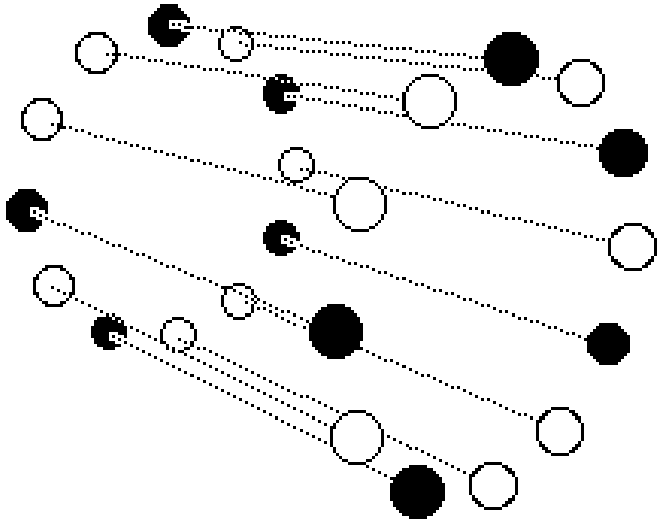
- Q: How many major 2nds does the pentatonic scale contain?



A: three!

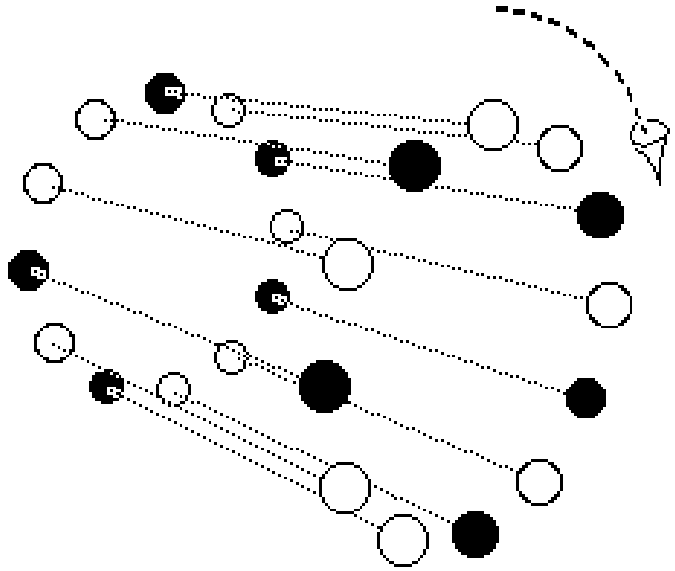
Another Approach

- Find the same answer this way
- Line up identical copies...
- ...and then turn the front one



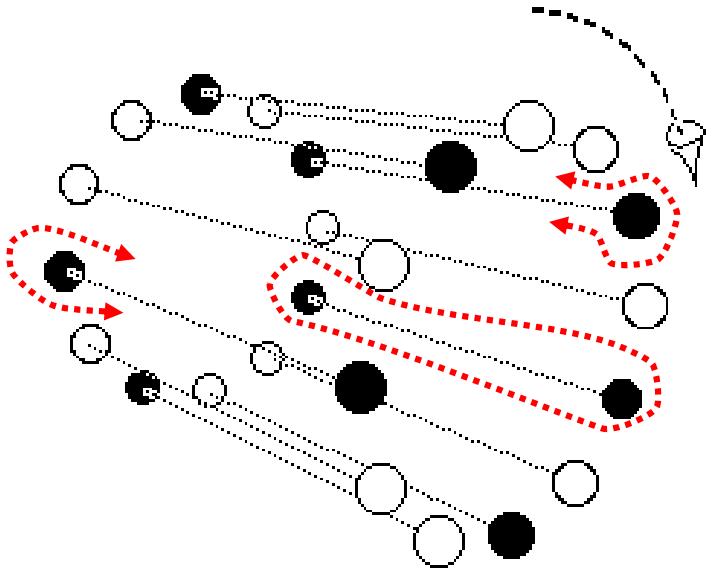
Autocorrelation

- Count the matching notes
- Each match represents a M2 interval in the scale



Autocorrelation

- Count the matching notes
- 3 matches ⇒ ∃ 3 M2 intervals in the pentatonic scale
- Should say M2/m7



Interval Spectrum

- Pentatonic contains:
  - 5 unisons (trivial)
  - 0 m2/M7
  - 3 M2/m7
  - 2 m3/M6
  - 1 M3/m6
  - 4 P4/P5
  - 0 ♭5

All different!
This is unique

Maximum for all 5-scales!

Diatonic Scale

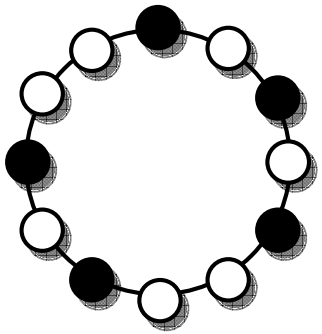
- Diatonic contains:
  - 7 unisons (trivial)
  - 2 m2/M7
  - 5 M2/m7
  - 4 m3/M6
  - 3 M3/m6
  - 6 P4/P5
  - 1 ♭5

All different!
This is unique

Maximum for all 7-scales!

More Properties

- This pattern is special
- What other properties does it have?



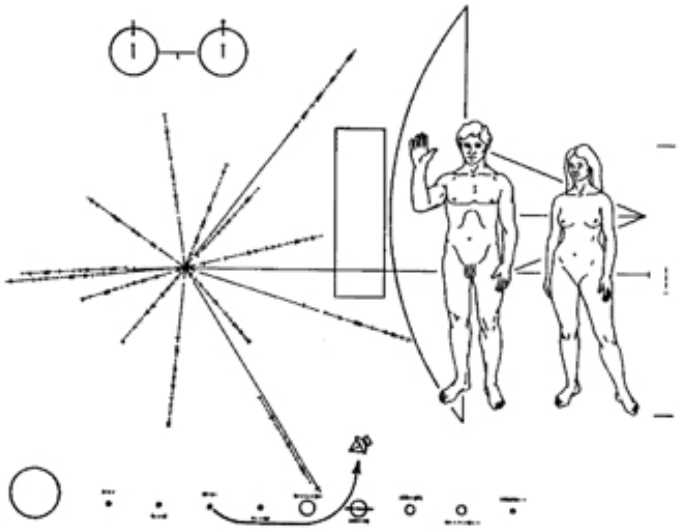
Your idea here

Some Ideas

- Local neighborhood
  - Figure out from a small neighborhood where you are in the scale
- Entropy
  - Define some sort of entropy on a scale

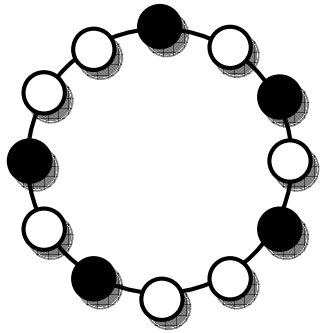
Intelligence

- How to prove we are smart?
- Pattern should have been on Voyager spacecraft



2π = 110.010010000111111011010101000 1...

e = 10.101101111110000101010001011 00...



Hvala!

- More details at http://www.andrewduncan.ws/cmt
- http://www.andrewduncan.ws/zboard/aes92preprint
- http://www.andrewduncan.ws/air